Normal gradient error and normal quadrupole correctors

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1 Effect of gradient errors

We address here the effect of gradient error and the general requirement for correction of the gradient error induced resonances. Gradient error in quadrupoles creates two main effect: 1) a detuning $\delta Q_x, \delta Q_y$; 2) an half integer stop-band $\delta Q_{x,sp}, \delta Q_{y,sp}$. The expression for $\delta Q_x, \delta Q_y, \delta Q_{x,sp}, \delta Q_{y,sp}$ for one error seed are given in [1]. We consider here a shift of the gradient component δk of a quadrupole gradient strength k. The j-th quadrupole that has strength k in m^{-2} will have a deviation δk_j. We take the relative error $\delta k_j/k_j$ as a measure of the gradient error. We consider the distribution of $\delta k_j/k_j$ of each quadrupole in SIS100 to be a Gaussian variable of average zero and standard deviation ξ. That means that we consider here random fluctuation of the gradient that are of the same type for each quadrupole. By using this general ansatz we find that the standard deviation of the detuning $\delta Q_{x/y}$ is

$$\sigma_{\delta Q_{x/y}} = \frac{1}{4\pi}\sqrt{\sum_j \overline{\beta_{x/y,j}}^2 k_{x/y,j}{}^2 \Delta^2 s_j} \quad \xi$$

where $\overline{\beta_{x/y,j}}$ is the average x/y beta function in the j-th quadrupole; $k_{x/y,j}$, is the x/y strength of the j-th quadrupole; $\Delta^2 s_j$ is the length of the j-th quadrupole, and ξ is the rms relative gradient error in each quadrupole. The standard deviation of the gradient half integer stop-band $\delta Q_{x/y,sp}$ is

$$\sigma_{\delta Q_{x/y,sp}} \leq \sqrt{\langle \delta Q^2_{x/y,sb}\rangle} = \frac{1}{2\pi}\sqrt{\sum_j \overline{\beta_{x/y,j}}^2 k^2_{x/y,j} \Delta s_j^2} \quad \xi$$

Simulation tests have shown that these formulas are correct.

2 Gradient error in SIS100: tolerable errors

For SIS100 in the Uranium scenario these two expression yields

$$\sigma_{\delta Q_{x/y}} = 4.1\xi \quad (1)$$

$$\sigma_{\delta Q_{x/y,sp}} \leq 8.2\xi \quad (2)$$

The gradient random errors tolerable for quadrupole has to limit the damage in terms of detuning and stop-band. An arbitrary choice is to consider the maximum acceptable detuning produced by the random error in gradient to be not larger than 10^{-3}. This value is also taken by the observation that in control rooms, the tune settings is tunable up to 3 digits after comma. By using Eq. 1 we find

$$\xi = 2.4 \times 10^{-4}$$

In the simulations presented in [2] the "reasonable seed" created a half integer stop-band of $\sigma_{\delta Q_{x/y,sp}} \simeq 0.01$ from which applying Eq. 2 we obtain $\xi \simeq 4 \times 10^{-4}$. **We suggest therefore the random error of the gradient to be** 2×10^{-4}

3 Strength of the correctors

The strength of the normal corrector can be judged with a conservative argument. Each quadrupole as strength $k \simeq 0.21$ m^{-2}. assuming that all the fluctuations are accumulated into one quadrupole we would find for $\xi = 2 \times 10^{-4}$ a maximal strength of

$$\text{max gradient err.} = 168 \quad \text{quadrupoles} \times k \times \xi \times 3$$

that is

$$\text{max gradient err.} = 168 \times 0.21 \times 2 \times 10^{-4} \times 3 = 0.021 \quad \text{m}^{-2}.$$

For a worse case scenario with $\xi = 1.2 \times 10^{-3}$ we find

$$\text{max gradient err.} = 0.13 \quad \text{m}^{-2}.$$

Both these numbers, 0.021, 0.13 m^{-2} should be compared with the integrated strength of the foreseen quadrupoles correctors in SIS100. The maximum strength of the corrector quadrupole is 0.75 T/m, corresponding at injection energy of 200 MeV/u i.e. for $B\rho = 1.8$ Tm at the maximum strength of 0.41 m^{-2}. Therefore the total integrated power of the corrector quadrupoles is $12 \times 0.41 = 4.92$ m^{-2} which is certainly larger than the total integrated strength of the gradient errors 0.13 m^{-2} (worse case). **The actual normal quadrupole corrector strength is safe with respect to the integrated strength of the gradient errors.**

4 Number of correctors for resonance compensation

Each normal resonance needs 2 correctors properly de-phased. The gradient error induced half integer resonances have to be compensated "simultaneously". The compensation of only one resonance easily may excite another near by resonance. As the working point is located in a quadrant of size 0.25, there are 4 resonances that have to be controlled simultaneously. For SIS100 these resonances are $2Q_x = 38, 2Q_x = 37, 2Q_y = 38, 2Q_y = 37$, but in general for any other choice of a working point, there are always 4 half integer resonances that surround it. Therefore 4×2 normal quadrupole correctors is the minimum to be used. Adding 2 quadrupole correctors for safety this leads to 10 correctors. **Considering the symmetry of the machine the natural choice is to take 2 quadrupoles per period.**

References

[1] E. D. Courant and H. S. Snyder, Ann. Phys. **3**, 1 (1958), ann. Phys. (N.Y.) 3 (1958) 1-48. In *Pellegrini, C. (ed.) et al.: The development of colliders* 23-70

[2] G. Franchetti and S. Sorge, Proc. of IPAC2013, pag. 1556(2013)