Influence of random tilts of quadrupole magnets on the closed orbit distortion in SIS100

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1 Aim and assumptions

Recent measurements of the alignment of SIS100 quadrupoles in their doublets revealed tilts of the quadrupole magnets of the order of magnitude ~ 0.1 mrad [1]. That led to the question of the influence of such tilts on the beam and, in particular, on the closed orbit distortion (COD) in SIS100.

The aim of this study is to estimate the contribution to the COD arising from these tilts and to compare it to those caused by other sources which so far have been taken into into account. "Tilt" in this document means both of the rotations of the magnets around the vertical axis $\Delta\theta$ and the horizontal axis $\Delta\phi$ at the longitudinal position of the half length.

The study is done for the nominal working point for fast extraction of heavy ions, i.e. $Q_x = 18.84$ and $Q_y = 18.73$. The sources of COD in SIS100 taken into account in previous studies are

- random shifts Δx and Δy of the quadrupoles.
- random roll angles $\Delta\psi$ of the dipole magnets, i.e. rotation around the longitudinal magnet axis.
- random deviations $\Delta\alpha$ of the dipoles' deflection angles.

All these magnet errors are assumed to follow Gaussian distributions truncated at 2σ. The widths of the Gaussian distributions are

- $\sigma_{\Delta x} = \sigma_{\Delta y} = 1$ mm for the quadrupoles.
- $\sigma_{\Delta\psi} = 4.04$ mrad for the dipoles which arises from the assumption that the position of left and right edges of the dipoles can independently vary with an rms uncertainty of $\sigma_{\text{dipole corner}} = 1$ mm and a transverse dipole magnet width of $w_{\text{dipole}} = 0.35$ m.
- $\sigma_{\Delta\alpha}/|\alpha| = 4.0 \cdot 10^{-3}$ for the dipoles, where the unperturbed deflection angle is $\alpha = -3.33333^0 = -58.178$ mrad.

The assumed rms widths of the tilt angles are $\sigma_{\Delta\theta} = \sigma_{\Delta\phi} = 0.5$ mrad.

2 Results

For this study, COD calculations are performed using the MAD-X code. 1000 samples of magnet errors are applied for the three scenarios

- with taking into account only the tilts of the quadrupoles.
- with taking into account only the COD sources of previous studies.
- with taking into account the COD sources of previous studies and the tilts of the quadrupoles.

In order to characterise the COD, the maximum COD $z_{\mathrm{co,max}}$ and the ring rms value

$$z_{\mathrm{co,rms}} \equiv \sqrt{\frac{1}{C} \oint \mathrm{d}s \; z_{\mathrm{co}}^2(s)} \tag{1}$$

are extracted for $z = x, y$ from the MAD-X output for each sample of magnet errors. Afterwards, the ensemble average,

$$\langle z_{\mathrm{co}} \rangle_{\mathrm{ensemble}} \equiv \frac{1}{N_{\mathrm{seeds}}} \sum_{n=1}^{N_{\mathrm{seeds}}} z_{\mathrm{co}}(n), \tag{2}$$

and $z_{\mathrm{co}}(0.99)$ which is the maximum of the lowest 99 % and defined by

$$\int\limits_{0}^{z_{\mathrm{co}}(0.99)} \mathrm{d}z_{\mathrm{co}} \; f(z_{\mathrm{co}}) = 0.99 \tag{3}$$

are determined, where the distribution function $f(z_{\mathrm{co}})$ in the last equation is normalised to unity. $z_{\mathrm{co}}(0.99)$ is chosen instead of the maximum z_{co} found among all error samples in order to gain statistical confidence. These characteristics are presented in Table 1.

The characteristics of the COD caused only by the quadrupole tilts is about 0.2 % of the corresponding characteristics of the COD obtained only for the magnet errors used in previous studies. There is not a clear tendency of the change if the quadrupole tilts are in addition taken into account. That suggests that the tilts of the quadrupoles do not play a role for the size of the COD in SIS100.

References

[1] G. J. Ketter, private communication.

Table 1: Characteristics of the closed orbit distortion found only with tilts of the quadrupoles in Table A, only with the magnet errors applied in previous studies in Table B, and with magnet errors applied in previous studies and quadrupoles tilts in Table C.

Table A

$x_{\mathrm{co,max}}(0.99)$	3.17 mm
$x_{\mathrm{co,rms}}(0.99)$	1.52 mm
$y_{\mathrm{co,max}}(0.99)$	2.16 mm
$y_{\mathrm{co,rms}}(0.99)$	0.925 mm
$\langle x_{\mathrm{co,max}} \rangle_{\mathrm{ensemble}}$	1.61 mm
$\langle x_{\mathrm{co,rms}} \rangle_{\mathrm{ensemble}}$	0.684 mm
$\langle y_{\mathrm{co,max}} \rangle_{\mathrm{ensemble}}$	1.17 mm
$\langle y_{\mathrm{co,rms}} \rangle_{\mathrm{ensemble}}$	0.435 mm

Table B

$x_{\mathrm{co,max}}(0.99)$	135 mm
$x_{\mathrm{co,rms}}(0.99)$	64.4 mm
$y_{\mathrm{co,max}}(0.99)$	91.7 mm
$y_{\mathrm{co,rms}}(0.99)$	39.2 mm
$\langle x_{\mathrm{co,max}} \rangle_{\mathrm{ensemble}}$	70.9 mm
$\langle x_{\mathrm{co,rms}} \rangle_{\mathrm{ensemble}}$	29.8 mm
$\langle y_{\mathrm{co,max}} \rangle_{\mathrm{ensemble}}$	51.9 mm
$\langle y_{\mathrm{co,rms}} \rangle_{\mathrm{ensemble}}$	19.1 mm

Table C

$x_{\mathrm{co,max}}(0.99)$	134 mm
$x_{\mathrm{co,rms}}(0.99)$	64.8 mm
$y_{\mathrm{co,max}}(0.99)$	92.2 mm
$y_{\mathrm{co,rms}}(0.99)$	39.0 mm
$\langle x_{\mathrm{co,max}} \rangle_{\mathrm{ensemble}}$	70.9 mm
$\langle x_{\mathrm{co,rms}} \rangle_{\mathrm{ensemble}}$	29.8 mm
$\langle y_{\mathrm{co,max}} \rangle_{\mathrm{ensemble}}$	52.0 mm
$\langle y_{\mathrm{co,rms}} \rangle_{\mathrm{ensemble}}$	19.1 mm